

# Accretion of Phantom Energy and Generalized Second Law of Thermodynamics for Einstein-Maxwell-Gauss-Bonnet Black Hole

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## Abstract

We have investigated the accretion of phantom energy onto a 5-dimensional extreme Einstein-Maxwell-Gauss-Bonnet (EMGB) black hole. It is shown that the evolution of the EMGB black hole mass due to phantom energy accretion depends only on the pressure and density of the phantom energy and not on the black hole mass. Further we study the generalized second law of thermodynamics (GSL) at the event horizon and obtain a lower bound on the pressure of the phantom energy.

**Keywords:** Accretion; Einstein-Maxwell-Gauss-Bonnet black hole; phantom energy; dark energy; generalized second law of thermodynamics.

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## I. INTRODUCTION

Various astronomical observations convincingly show [1] that our universe is presently undergoing a phase of accelerated expansion. Within the framework of General Relativity (GR), the accelerated expansion of the universe could be explained by the presence of a ‘cosmological constant’ bearing negative pressure which results in the stretching of the spacetime [2]. Many other theoretical models have also been constructed to explain the accelerated expansion of the universe including based on homogeneous and time dependent scalar field like the quintessence [3], Chaplygin gas [4] and phantom energy [5], to name a few. The equation of state  $p = \omega\rho$ , with  $\omega < -1$ , characterizes the phantom energy. It possesses some weird properties: the cosmological parameters like scale factor and energy density become infinite in a finite time; all gravitationally bound objects lose mass with the accretion of phantom energy; the fabric of spacetime is torn apart at the big rip; and that it violates the standard relativistic energy conditions. The astrophysical data coming from the microwave background radiation categorically favors the phantom energy [6]. Motivated from the dark energy models, we model phantom energy by an ideal fluid with negative pressure.

Babichev et al [7] have shown that the mass of the black hole will decrease with time when we consider the accretion of phantom energy. They showed that the mass will vanish before the Big Rip. After this seminal work the accretion of dark energy onto a black hole have been investigated by many authors. In GR, the accretion of phantom energy onto Schwarzschild [7, 8], Reissner-Nordström (RN) [9], Kerr-Newman (KN) [10] and primordial black holes has been studied [11]. In the case of RN black hole, the mass of the black hole decreases but the electric charge remains unaffected. Consequently the naked singularity appears at the Big Rip. This henceforth violates the Penrose Cosmic Censorship Hypothesis which forbids the existence of naked singularities. This result also arises for the KN black hole. In another paper [12], the authors have investigated the accretion of phantom energy on galaxies and deduce their destruction due to phantom energy. The accretion of phantom energy on a 3-dimensional Banados-Teitelboim-Zanelli (BTZ) black hole was investigated in [13]. It was speculated there to investigate the accretion dynamics in higher dimensional gravities and modified theories of gravity.

The Einstein theory of gravity with Gauss-Bonnet (GB) term (given in the next section)

has some notable properties (see for example [14, 15]). The GB term attains nontrivial physical meaning in 5 dimensions [14]. Consequently we investigate the accretion of exotic phantom energy onto a static 5-dimensional EMGB black hole. We show that the expression of the evolution of EMGB black hole mass is independent of its mass and depends only on the energy density and pressure of the phantom energy. It is well-known that the horizon area of the black hole decreases with the accretion of phantom energy [16], hence it is essential to study the GSL in this case. We show that the validity of GSL in the present model yields a lower bound on the phantom energy pressure. Beside, we demonstrate that the first law of thermodynamics holds in the present construction.

The plan of the paper is as follows: In second section we model the accretion of phantom energy onto 5-dimensional EMGB black hole. In third section, we study the GSL for EMGB black hole. Finally we conclude our results.

## II. MODEL OF ACCRETION

The action in 5-dimensional spacetime  $(\mathfrak{M}, g_{\mu\nu})$  that represents the Einstein-Maxwell theory with a GB term and a cosmological constant has the expression [17, 18]

$$S = \frac{1}{2} \int d^5x \sqrt{-g} \left[ R - 2\Lambda - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \alpha R_{GB} \right], \quad (1)$$

where  $R_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\sigma\delta}R^{\mu\nu\sigma\delta}$ , is the GB term, while  $\alpha$  is the GB coupling parameter having dimension of  $L^2$  ( $\alpha^{-1}$  is related to the string tension in Heterotic string theory),  $\Lambda$  is a cosmological constant and  $F_{\mu\nu}$  is the electromagnetic field tensor. Variation of the action (1) with respect to the metric tensor yields the EMGB field equations

$$G_{\mu\nu} - \alpha H_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}, \quad (2)$$

where

$$H_{\mu\nu} = 2(RR_{\mu\nu} - 2R_{\mu\lambda}R_{\nu}^{\lambda} - 2R^{\gamma\delta}R_{\gamma\mu\delta\nu} + R_{\mu}^{\sigma\gamma\delta}R_{\sigma\nu\gamma\delta}) - \frac{1}{2}g_{\mu\nu}R_{GB}.$$

The spherically symmetric metric of a 5-dimensional EMGB black hole is [17, 19]

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_3^2, \quad (3)$$

where

$$d\Omega_3^2 = (d\theta_1^2 + \sin^2\theta_1(d\theta_2^2 + \sin^2\theta_2 d\theta_3^2))$$

and

$$f(r) = 1 + \frac{r^2}{4\alpha} - \frac{r^2}{4\alpha} \sqrt{1 + \frac{16M\alpha}{\pi r^4} - \frac{8Q^2\alpha}{3r^6} + \frac{4\Lambda\alpha}{3}}, \quad (4)$$

$M$  is the mass and  $Q$  is the charge of the black hole. For extreme EMGB black hole  $M = Q$ . The coefficient  $g_{00}$  is termed as the lapse function. The event horizon of the extreme EMGB black hole is obtained by setting  $f(r) = 0$ , and  $df(r)/dr = 0$  which turns out [17]

$$r_e^2 = \frac{1}{\Lambda} \left(1 + 2 \cos \frac{\beta}{3}\right), \quad (5)$$

where

$$\cos \beta = 1 - \frac{Q^2 \Lambda^2}{2}, \quad Q^2 \Lambda^2 < 4. \quad (6)$$

Also we have  $\sqrt{|g|} = r^3 \sin^2 \theta_1 \sin \theta_2$ , where  $g$  is the determinant of the metric. Here the apparent horizon is defined by  $A = 4\Omega_4 r^3$ , where  $\Omega_4 = \pi^2/\Gamma(3)$  [15].

To analyze the accretion of phantom energy onto the EMGB black hole, we employ the formalism from the work by Babichev et al [7]. The stress energy momentum tensor representing the phantom energy is the perfect fluid

$$T^{\mu\nu} = (\rho + p)u^\mu u^\nu + pg^{\mu\nu}, \quad (7)$$

where  $\rho$  and  $p$  are the energy density and pressure of the phantom energy while  $u^\mu = (u^0, u^1, 0, 0, 0)$  is the velocity five vector of the fluid flow. Also  $u^1 = u$  is the radial velocity of the flow while the components  $u^2$ ,  $u^3$  and  $u^4$  are zero due to spherical symmetry of the EMGB black hole. Using the energy-momentum conservation for  $T^{\mu\nu}$  i.e.  $T_{;\mu}^{\mu\nu} = 0$ , we obtain

$$u\sqrt{|g|}(\rho + p)\sqrt{f(r) + u^2} = C_1, \quad (8)$$

where  $C_1$  is constant of integration. Since the flow is inwards the black hole therefore  $u < 0$ . Also the projection of the energy momentum conservation along the velocity five vector  $u_\nu T_{;\mu}^{\mu\nu} = 0$  (the energy flux equation) is

$$u\sqrt{|g|} \exp \left[ \int_{\rho_\infty}^{\rho_h} \frac{d\rho}{\rho + p} \right] = -A_1. \quad (9)$$

Here  $A_1$  is an integration constant and the associated minus sign is taken for convenience. Also  $\rho_h$  and  $\rho_\infty$  are the energy densities of phantom energy at the EMGB horizon and at

infinity respectively. From (8) and (9), we obtain

$$(\rho + p)\sqrt{f(r) + u^2} \exp \left[ - \int_{\rho_\infty}^{\rho_h} \frac{d\rho}{\rho + p} \right] = C_2, \quad (10)$$

where  $C_2 = -C_1/A_1 = \rho_\infty + p(\rho_\infty)$ . The rate of change in the mass of black hole  $\dot{M} = -2\pi^2 r^3 T_0^1$ , is given by

$$dM = 2\pi^2 A_1 (\rho_\infty + p_\infty) dt. \quad (11)$$

Note that  $\rho_\infty + p_\infty < 0$  (violation of null energy condition) leads to decrease in the mass of the black hole. Moreover, the above expression is also independent of mass contrary to the Schwarzschild, Reissner-Nordström and Kerr-Newman black holes [8–10]. Further, the last equation is valid for any general  $\rho$  and  $p$  violating the null energy condition, thus we write

$$dM = 2\pi^2 A_1 (\rho + p) dt. \quad (12)$$

### III. GENERALIZED SECOND LAW OF THERMODYNAMICS AND EMGB BLACK HOLE

In this section we discuss the thermodynamic of phantom energy accretion that crosses the event horizon of the EMGB black hole. Let us first write the EMGB metric in the form

$$ds^2 = h_{mn} dx^m dx^n + r^2 d\Omega_3^2, \quad m, n = 0, 1 \quad (13)$$

where  $h_{mn} = \text{diag}(-f(r), 1/f(r))$ , is a 2-dimensional metric. From the condition of normalized velocities  $u^\mu u_\mu = -1$ , we obtain the relations

$$u^0 = f(r)^{-1} \sqrt{f(r) + u^2}, \quad u_0 = -\sqrt{f(r) + u^2}. \quad (14)$$

The components of stress energy tensor are

$$T^{00} = f(r)^{-1} [(\rho + p) \left( \frac{f(r) + u^2}{f(r)} \right) - p], \quad (15)$$

and

$$T^{11} = (\rho + p)u^2 + f(r)p. \quad (16)$$

With the help of (15) and (16) we calculate the work density which is defined by  $W = -\frac{1}{2} T^{mn} h_{mn}$  [15]. It comes out

$$W = \frac{1}{2} (\rho - p). \quad (17)$$

The energy supply vector is defined by

$$\Psi_n = T_n^m \partial_m r + W \partial_n r. \quad (18)$$

The components of the energy supply vector are

$$\Psi_0 = T_0^1 = -u(\rho + p) \sqrt{f(r) + u^2}, \quad (19)$$

and

$$\Psi_1 = T_1^1 + W = (\rho + p) \left( \frac{1}{2} + \frac{u^2}{f(r)} \right). \quad (20)$$

The change of energy across the apparent horizon is determined through  $-dE \equiv -A\Psi$ , where  $\Psi = \Psi_0 dt + \Psi_1 dr$ . The energy crossing the event horizon of the EMGB black hole is given by

$$dE = 2\pi^2 r_e^3 u^2 (\rho + p) dt. \quad (21)$$

Assuming  $E = M$  and comparing (12) and (21), we can determine the value of constant  $A_1 = u^2 \sqrt{|g|}$ .

The entropy of the EMGB black hole is [15]

$$S_h = \frac{\pi^2 r_e^3}{2} \left( 1 + \frac{12\alpha}{r_e^2} \right). \quad (22)$$

It can be shown easily that the thermal quantities, change of phantom energy  $dE$ , horizon entropy  $S_h$  and horizon temperature  $T_h$  satisfy the first law  $dE = T_h dS_h$ , of thermodynamics. After differentiation of last equation w.r.t.  $t$ , and using (12), we have

$$\dot{S}_h = \pi u^2 (\rho + p) r_e^3. \quad (23)$$

Since all the parameters are positive in (23) except that  $\rho + p < 0$ , it shows that the second law of thermodynamics is violated i.e.  $\dot{S}_h < 0$ , as a result of accretion of phantom energy on the EMGB black hole.

Now we proceed to the GSL. It is defined by [20]

$$\dot{S}_{tot} = \dot{S}_h + \dot{S}_{ph} \geq 0. \quad (24)$$

In other words, the sum of the rate of change of entropies of black hole horizon and phantom energy must be positive. We consider event horizon of the EMGB black hole as a boundary of thermal system and the total matter energy within the event horizon is the mass of the

EMGB black hole. We also assume that the horizon temperature is in equilibrium with the temperature of the matter-energy enclosed by the event horizon, i.e.  $T_h = T_{ph} = T$ , where  $T_{ph}$  is the temperature of the phantom energy. Similar assumptions for the temperatures  $T_h$  and  $T_{ph}$  has been studied in [21]. We know that the Einstein field equations satisfy first law of thermodynamics  $T_h dS_h = pdV + dE$ , at the event horizon [22]. We also assume that the matter-energy enclosed by the event horizon of the EMGB black hole also satisfy the first law of thermodynamics given by

$$T_{ph} dS_{ph} = pdV + dE. \quad (25)$$

Here the horizon temperature is given by [15]

$$T_h = \frac{1}{2\pi r_e}. \quad (26)$$

In this paper, we are assuming that  $T_h = T_{ph} = T$ . Therefore (24) gives

$$T\dot{S}_{tot} = T(\dot{S}_h + \dot{S}_{ph}) = 4\pi^2 u^2 (\rho + p) r_e^3 (1 - p\pi^2 r_e^2 k(r_e)), \quad (27)$$

where

$$k(r_e) = -\frac{\sqrt{1 - (\frac{r_e^2 \Lambda - 1}{2})^2}}{3\sqrt{4 - \Lambda^2 Q^2}}. \quad (28)$$

From the above equation, note that  $u^2 > 0$ ,  $r_e^3 > 0$  and  $\rho + p < 0$ . The GSL holds provided  $1 - p\pi^2 r_e^3 k(r) < 0$  which implies

$$p > \frac{1}{\pi^2 r_e^3 k(r_e)}. \quad (29)$$

Since the pressure of the phantom energy is negative ( $p < 0$ ), therefore the GSL gives us the lower bound on the pressure of the phantom energy.

$$-\frac{3\sqrt{4 - \Lambda^2 Q^2}}{\pi^2 r_e^2 \sqrt{1 - (\frac{r_e^2 \Lambda - 1}{2})^2}} < p < 0. \quad (30)$$

The GSL in the phantom energy accretion holds within the inequality (30) which is independent of the GB parameter. Otherwise the GSL does not hold which forbid evaporation of EMGB black hole by the phantom accretion [23]. In addition, it is not clear whether the GSL should be valid in presence of the phantom fluid not respecting the dominant energy condition [23].

#### IV. CONCLUSION

In this paper, we have studied the accretion of exotic phantom energy onto extreme EMGB black hole. The motivation behind this work is to investigate the accretion dynamics in higher dimensional gravity. Our analysis has shown that evolution of mass of the EMGB black hole would be independent of its mass and will be dependent only on the energy density and pressure of the phantom energy in its vicinity. Due to spherical symmetry, the accretion process is simple since the phantom energy falls radially on the black hole. This result is similar to the one obtained for the BTZ black hole [13]. Since the result of the accretion of phantom energy in 4-dimensions (for the Schwarzschild, RN and KN black holes) is mass dependent and in the case of 3-dimension BTZ and 5-dimensional EMGB is mass independent. This raises the question whether this dependency of  $\dot{M}$  on mass is restricted to black holes in 4-dimensions only. Therefore we conjecture the following: *the rate of change of mass of a black hole due to the phantom energy depends on the mass of a black hole in 4-dimensions only.*

We also studied GSL for the EMGB black hole. This is performed on the assumption that the event horizon of EMGB black hole acts as a boundary of the thermal system. Moreover the phantom energy crosses the event horizon radially and reduces the mass of the black hole. Furthermore the black hole horizon is in thermal equilibrium with the phantom energy falling on the black hole. Under these assumptions we deduced that the GSL holds provided the pressure of the phantom energy  $p$  acts as the lower bound (30) on the black hole parameters.

An interesting question is that what would be the fate of the black hole near the big rip. For a Schwarzschild black hole [7], the mass completely disappears without any remnant, while for the Reissner-Nordström black hole [9], a remnant in the form of naked singularity appears because the accretion does not effect the charge. In the present scenario of an extremal EMGB black hole, for which mass and charge are on equal footing, the accretion leaves *no* remnant analogous to a Schwarzschild black hole. We emphasize that the present study cannot be reduced to that for Schwarzschild solution by choosing  $\alpha = 0$  since the EMGB solution (4) becomes undefined.



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